[nex94] Probabilities of complements, unions, and intersections

Given are the probability axioms (i) $P(A) \geq 0$, (ii) $P(S) = 1$, and (iii) $P(A + B) = P(A) + P(B)$ if $AB = \emptyset$, where S is the sample space and A, B are events.

Derive from these axioms the following simple theorems regarding complements, unions, and intersections of events:

(a) $P(\emptyset) = 0$, (b) $P(\bar{A}) = 1 - P(A)$, (c) $P(A + B) = P(A) + P(B) - P(AB)$.

The notions $A + B$, AB, $\bar{A}$, $\emptyset$, mean union, intersection, complement, and empty set, respectively.

Solution: