[nex90] Conditional probability

Given the simple probabilites $P(A), P(B)$ of two events A, B and their joint probability $P(AB)$ defined as the probability of their intersection, the conditional probabilities $P(A|B), P(B|A)$ are then defined via

$$P(AB) = P(A|B)P(B) = P(B|A)P(A).$$

Show that the conditional probability $P(A|B) = P(AB)/P(B)$ is indeed a probability in the formal sense by showing that it satisfies the probability axioms: (i) $P(A|B) \geq 0$, (ii) $P(S|B) = 1$, (iii) $P(A + C|B) = P(A|B) + P(C|B)$ if $AC = \emptyset$.

Solution: