[nex70] Critically damped ballistic galvanometer.

The response of a critically damped ballistic galvanometer to a current pulse at $t = 0$ is $\Psi(t) = cte^{-\gamma t}$. Consider the situation where the galvanometer experiences a steady stream of independent random current pulses, $X(t) = \sum_k \Psi(t-t_k)$, where the t_k are distributed randomly with an average rate n of occurrences.
(a) Use Campbell's theorem [nex37] to calculate the average displacement $\langle X \rangle$ and the autocorrelation function $\langle\langle X(t)X(0)\rangle\rangle$.
(b) Show that the associated spectral density reads

$$S_{XX}(\omega) \doteq \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle\langle X(t)X(0)\rangle\rangle = \frac{nc^2}{(\gamma^2+\omega^2)^2}.$$

Solution: