[nex67] Linear response of classical oscillator.

The classical oscillator is defined by the equation of motion,

$$m\ddot{x} + \gamma\dot{x} + m\omega_0^2 x = a(t), \tag{1}$$

where γ is the attenuation coefficient, $m\omega_0^2$ the spring constant, and $a(t)$ a weak periodic perturbation. The (linear) response function is defined by the relation

$$\langle x(t)\rangle - \langle x\rangle_0 = \int_{-\infty}^{t} dt' \tilde{\chi}_{xx}(t-t')a(t'), \tag{2}$$

where $x(t)$ is the solution of (1).
(a) Calculate the generalized susceptibility $\chi_{xx}(\omega)$ as well as its reactive part $\chi'_{xx}(\omega)$ and its dissipative part $\chi''_{xx}(\omega)$.
(b) Use the (classical) fluctuation-dissipation theorem to infer the spectral density $\Phi_{xx}(\omega)$ from the dissipation function $\chi''_{xx}(\omega)$.

Solution: