[nex56] Mean-square displacement of Brownian particle I

Consider a Brownian particle of mass m constrained to move along a straight line. The particle experiences two forces: a drag force $-\gamma v$ and a white-noise random force $f(t)$. In [nex118] we inferred from the Langevin equation an ODE for the mean-square displacement and solved it to obtain

$$\langle x^2(t)\rangle = 2D\left[t - \frac{m}{\gamma}\left(1 - e^{-(\gamma/m)t}\right)\right].$$

Here the task is to calculate $\langle x^2(t)\rangle$ from the (steady-state) velocity autocorrelation function,

$$\langle v(t_1)v(t_2)\rangle = \frac{k_B T}{m} e^{-(\gamma/m)|t_1 - t_2|}$$

determined in [nex55], via integration with initial condition $x(0) = 0$.

Solution: