[nex5] Random chords (Bertrand's paradox)

Consider a circle of unit radius and draw $at\ random$ a straight line intersecting it in a chord of length L

(a) by taking lines through an arbitrary fixed point on the circle with random orientation;

(b) by taking lines perpendicular to an arbitrary diameter of the circle with the point of intersection chosen randomly on the diameter;

(c) by choosing the midpoint of the chord at random in the area enclosed by the circle.

For each random choice determine the probability distribution P(L) for the length of the chord and calculate the average length $\langle L \rangle$.

Solution: