[nex27] Specifications of diffusion process

Examine the diffusion process,

$$P(x|x_0; \Delta t) = \frac{1}{\sqrt{4\pi D \Delta t}} \exp\left(-\frac{(x-x_0)^2}{4D\Delta t}\right),$$

as a special solution of the differential Chapman-Kolmogorov equation by determining the three specifications (two of which are zero):

$$W(x|x_0) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(x|x_0; \Delta t),$$

$$A(x) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|x-x_0|<\epsilon} dx (x-x_0) P(x|x_0; \Delta t), \quad B(x) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|x-x_0|<\epsilon} dx (x-x_0)^2 P(x|x_0; \Delta t).$$

Use the results for $W(x|x_0)$, $A(x)$, and $B(x)$ to simplify the differential Chapman-Kolmogorov equation from [nln56] nto the diffusion equation.

Solution: