[nex129] Brownian harmonic oscillator VII: equivalent specifications

In [nln75] we have introduced two alternative specifications for the Brownian harmonic oscillator:

$$m\ddot{x} + \gamma\dot{x} + kx = f_{\mathrm{w}}(t), \tag{1}$$

$$m\frac{dx}{dt} + \int_{-\infty}^{t} dt'\,\alpha(t-t')x(t') = \frac{1}{\omega_0}f_{\mathrm{c}}(t), \quad \alpha(t) = m\omega_0^2 e^{-(\gamma/m)t}, \tag{2}$$

where the white-noise random force $f_{\mathrm{w}}(t)$ and the correlated-noise random force $f_{\mathrm{c}}(t)$ each satisfy the fluctuation-dissipation relation introduced in [nln72]. Derive specification (1) from specification (2) including the change in random force.

Solution: