## [nex109] Air in leaky tank II: probability distribution

At time $t = 0$ a tank of volume $V$ contains $n_0$ molecules of air (disregarding chemical distinctions). The tank has a tiny leak and exchanges molecules with the environment, which has a constant density $\rho$ of air molecules.
(a) Infer from the generating function,

$$G(z,t) = e^{V\rho(z-1)\left[1-e^{-t/V}\right]}\left[e^{-t/V}(z-1)+1\right]^{n_0},$$

calculated in [nex48], via a power series expansion, the probability distribution,

$$P(n,t) = e^{-V\rho\left[1-e^{-t/V}\right]}\sum_{m=0}^{\min(n,n_0)}\frac{n_0!}{m!(n_0-m)!(n-m)!}(\rho V)^{n-m}(1-e^{-t/V})^{n+n_0-2m}e^{-mt/V},$$

for the number of molecules in the tank.
(b) Demonstrate consistency of this result with the initial condition: $\lim_{t\to 0} P(n,t) = \delta_{n,n_0}$.
(c) Show that the equilibrium state is described by a Poisson distribution by (i) expanding the equilibrium generating function $G(z,\infty)$ from [nex48] into a power series, and (ii) by taking the limit $P(n, t \to \infty)$.

**Solution:**