[nex102] House of the mouse: two-way doors only
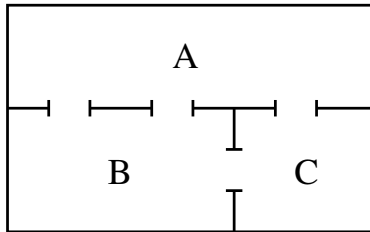
A trained mouse lives in a house with floor plan as shown. A bell rings at regular time intervals, prompting the mouse to go to an adjacent room through any door with equal probability.

(a) If the mouse starts from room A or room B or room C, with what probability is he in each room after two rings of the bell? Begin by constructing the transition matrix $\mathbf{W}$ and the initial vector $\vec{P}(0)$.

(b) Calculate the probability of the mouse being in each room after the bell has rung a great many times. Does it matter where the mouse was initially? Carry out this part in two ways: (i) Solve the left-eigenvector problem of matrix $\mathbf{W}$ to get the stationary distribution $\vec{\pi}$. (ii) Calculate the stationary distribution $\vec{\pi}$ directly from the detailed-balance condition.

Solution: