[nex1] Subtlety of statistical independence

(i) Given that events A, B are statistically independent, $P(AB) = P(A)P(B)$, show that the event pairs $\bar{A}, B$ and $\bar{A}, \bar{B}$ are also statistically independent: $P(\bar{A}B) = P(\bar{A})P(B),\ P(\bar{A}\bar{B}) = P(\bar{A})P(\bar{B})$.
(ii) Consider three events A, B, C occurring with probabilities $P(A), P(B), P(C)$, respectively, and satisfying the relations $P(AB) = P(A)P(B),\ P(BC) = P(B)P(C),\ P(CA) = P(C)P(A)$. Show that these relations are compatible with the relation $P(ABC) \neq P(A)P(B)P(C)$, in which case the three events are not statistically independent.

Solution: