Gram-Schmidt Orthogonalization I [nln83]

First three iterations in step #1 of [nln82].

Initial condition:

$$|f_0\rangle = A.$$

First iteration:

$$\imath L|f_0\rangle = |f_1\rangle \quad \Rightarrow \ \langle f_0|f_1\rangle = \langle f_0|\imath L|f_0\rangle = 0.$$

Second iteration:

$$\begin{aligned}
\imath L|f_1\rangle = |f_2\rangle - \Delta_1|f_0\rangle \quad &\Rightarrow \ \langle f_1|f_2\rangle = \underbrace{\langle f_1|\imath L|f_1\rangle}_{0} + \Delta_1 \underbrace{\langle f_1|f_0\rangle}_{0} = 0, \\
&\Rightarrow \ \langle f_0|f_2\rangle = \underbrace{\langle f_0|\imath L|f_1\rangle}_{-\langle f_1|f_1\rangle} + \Delta_1 \langle f_0|f_0\rangle = 0 \\
&\qquad\qquad \text{if } \Delta_1 = \frac{\langle f_1|f_1\rangle}{\langle f_0|f_0\rangle}.
\end{aligned}$$

Third iteration:

$$\imath L|f_2\rangle = |f_3\rangle - \Delta_2|f_1\rangle - \Gamma_2|f_0\rangle$$

$$\begin{aligned}
&\Rightarrow \ \langle f_2|f_3\rangle = \underbrace{\langle f_2|\imath L|f_2\rangle}_{0} + \Delta_2 \underbrace{\langle f_2|f_1\rangle}_{0} + \Gamma_2 \underbrace{\langle f_2|f_0\rangle}_{0} = 0, \\
&\Rightarrow \ \langle f_1|f_3\rangle = \underbrace{\langle f_1|\imath L|f_2\rangle}_{-\langle f_2|f_2\rangle} + \Delta_2 \langle f_1|f_1\rangle + \Gamma_2 \underbrace{\langle f_1|f_0\rangle}_{0} = 0 \\
&\qquad\qquad \text{if } \Delta_2 = \frac{\langle f_2|f_2\rangle}{\langle f_1|f_1\rangle}, \\
&\Rightarrow \ \langle f_0|f_3\rangle = \underbrace{\langle f_0|\imath L|f_2\rangle}_{-\langle f_1|f_2\rangle = 0} + \Delta_2 \underbrace{\langle f_0|f_1\rangle}_{0} + \Gamma_2 \langle f_0|f_0\rangle = 0 \\
&\qquad\qquad \text{if } \Gamma_2 = 0.
\end{aligned}$$