Moment Expansion [nln78]

Correlation function and structure function:

$$\tilde{S}_{AA}(t) \doteq \langle A(t)A\rangle - \langle A\rangle^2 = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\imath\omega t} S_{AA}(\omega) = \sum_{n=0}^{\infty} \tilde{M}_n \frac{(-\imath t)^n}{n!}.$$

Frequency moments: use $\dot{\tilde{S}}_{AA}(t) = \langle \dot{A}(t)A\rangle = (-\imath/\hbar)\langle [A(t),\mathcal{H}]A\rangle$.

$$\tilde{M}_n \doteq \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\omega^n S_{AA}(\omega) = \imath^n \left[\frac{d^n}{dt^n}\tilde{S}_{AA}(t)\right]_{t=0} = \hbar^{-n}\langle \underbrace{[[\cdots[[}_{n} A,\mathcal{H}],\mathcal{H}],\cdots,\mathcal{H}]A\rangle,$$

High-temperature limit $T \to \infty$:

$$\tilde{M}_{2k+1} = 0, \quad \tilde{M}_{2k} = \hbar^{-2k}\langle \underbrace{[\cdots[}_{k} A,\mathcal{H}],\cdots,\mathcal{H}]\,\underbrace{[\cdots[}_{k} A,\mathcal{H}],\cdots,\mathcal{H}]\rangle.$$

Classical limit $\hbar \to 0$: use $\dot{\tilde{S}}_{AA}(t) = \langle \dot{A}(t)A\rangle = \langle \{A(t),\mathcal{H}\}A\rangle$.

$$\tilde{M}_{2k+1} = 0, \quad \tilde{M}_{2k} = (-1)^k \langle \underbrace{\{\{\cdots\{\{}_{2k} A,\mathcal{H}\},\mathcal{H}\},\cdots,\mathcal{H}\}A\rangle,$$

Fluctuation function:

$$\tilde{\Phi}_{AA}(t) \doteq \frac{1}{2}\langle [A(t),A]_+\rangle - \langle A\rangle^2 = \sum_{k=0}^{\infty} \tilde{M}_{2k} \frac{(-\imath t)^{2k}}{(2k)!},$$

$$\tilde{M}_{2k} = \frac{1}{2\hbar^{2k}}\langle \underbrace{[[\cdots[[}_{2k} A,\mathcal{H}],\mathcal{H}],\cdots,\mathcal{H}]A]_+\rangle.$$

Dissipation function:

$$\tilde{\chi}''_{AA}(t) \doteq \frac{1}{2\hbar}\langle [A(t),A]\rangle = \hbar^{-1}\sum_{k=0}^{\infty} \tilde{M}_{2k+1} \frac{(-\imath t)^{2k+1}}{(2k+1)!},$$

$$\tilde{M}_{2k+1} = \frac{1}{2\hbar^{2k+1}}\langle \underbrace{[[\cdots[[}_{2k+1} A,\mathcal{H}],\mathcal{H}],\cdots,\mathcal{H}]A]\rangle.$$

Moment expansion not guaranteed to converge.
Convergence problem may be circumnavigated by recursion method.