Time Dependence of Expectation Values [nln77]

Quantum Hamiltonian system:

$$\begin{aligned}\langle A\rangle_t &= \mathrm{Tr}\{\rho(0)A(t)\} \qquad \text{(Heisenberg representation)}\\ &= \mathrm{Tr}\Big\{\rho(0)e^{\imath\mathcal{H}t/\hbar}A(0)e^{-\imath\mathcal{H}t/\hbar}\Big\} = \mathrm{Tr}\Big\{e^{-\imath\mathcal{H}t/\hbar}\rho(0)e^{\imath\mathcal{H}t/\hbar}A(0)\Big\}\\ &= \mathrm{Tr}\{\rho(t)A(0)\} \qquad \text{(Schroedinger representation)}\end{aligned}$$

- $\dfrac{dA}{dt} = \imath\hat{L}A = \dfrac{\imath}{\hbar}[\mathcal{H}, A]$ (Heisenberg equation)
- $\dfrac{d\rho}{dt} = -\imath\hat{L}\rho = \dfrac{1}{\imath\hbar}[\mathcal{H}, \rho]$ (quantum Liouville equation)
- $\hat{L} \doteq \dfrac{1}{\hbar}[\mathcal{H},\ ]$ (quantum Liouville operator)
- $[\ ,\ ]$ (commutator)

Classical Hamiltonian system:

$$\begin{aligned}\langle A\rangle_t &= \int d^nq\int d^np\,\rho\big(q_1,\ldots,q_n;p_1,\ldots,p_n;0\big)A\big(q_1,\ldots,q_n;p_1,\ldots,p_n;t\big)\\ &= \int d^nq\int d^np\,\rho\big(q_1,\ldots,q_n;p_1,\ldots,p_n;t\big)A\big(q_1,\ldots,q_n;p_1,\ldots,p_n;0\big)\end{aligned}$$

- $\dfrac{dA}{dt} = \imath\hat{L}A = -\{\mathcal{H}, A\}$ (Hamilton equation)
- $\dfrac{d\rho}{dt} = -\imath\hat{L}\rho = \{\mathcal{H}, \rho\}$ (classical Liouville equation)
- $\hat{L} \doteq \{\mathcal{H},\ \} = \imath\displaystyle\sum_{j=1}^{n}\left(\frac{\partial\mathcal{H}}{\partial q_j}\frac{\partial}{\partial p_j} - \frac{\partial\mathcal{H}}{\partial p_j}\frac{\partial}{\partial q_j}\right)$ (classical Liouville operator)
- $\{\ ,\ \}$ (Poisson bracket)