Generating function [nln48]

The generating function $G_X(z)$ is a representation of the characteristic function $\Phi_X(k)$ that is most commonly used, along with factorial moments and factorial cumulants, if the stochastic variable X is integer valued.

Definition: $G_X(z) \doteq \langle z^x \rangle$ with $|z| = 1$.

Application to continuous and discrete (integer-valued) stochastic variables:

$$G_X(z) = \int dx\, z^x P_X(x), \qquad G_X(z) = \sum_n z^n P_X(n).$$

Definition of factorial moments:

$$\langle X^m \rangle_f \doteq \langle X(X-1)\cdots(X-m+1)\rangle, \quad m \geq 1; \quad \langle X^0 \rangle_f \doteq 0.$$

Function generating factorial moments:

$$G_X(z) = \sum_{m=0}^{\infty} \frac{(z-1)^m}{m!} \langle X^m \rangle_f, \quad \langle X^m \rangle_f = \frac{d^m}{dz^m} G_X(z)\bigg|_{z=1}.$$

Function generating factorial cumulants:

$$\ln G_X(z) = \sum_{m=1}^{\infty} \frac{(z-1)^m}{m!} \langle\langle X^m \rangle\rangle_f, \quad \langle\langle X^m \rangle\rangle_f = \frac{d^m}{dz^m} \ln G_X(z)\bigg|_{z=1}.$$

Applications:

- ▷ Moments and cumulants of the Poisson distribution [nex16]
- ▷ Pascal distribution [nex22]
- ▷ Reconstructing probability distributions [nex14]