Bertrand's paradox [nln41]

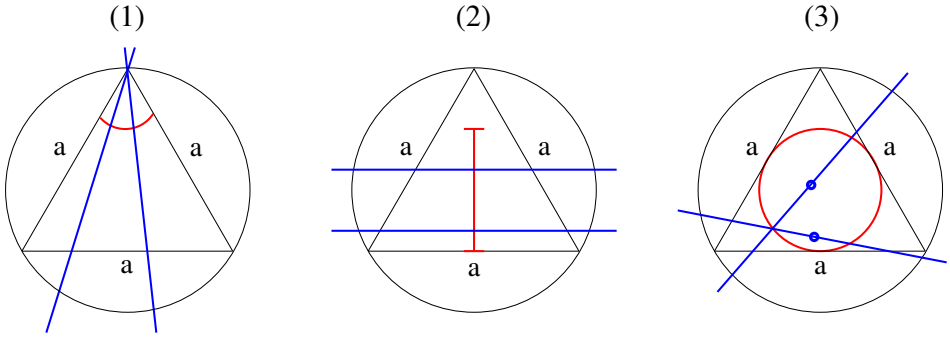
A circle of unit radius with an inscribed equilaterial triangle (of side $a = \sqrt{3}$) is painted on a flat horizontal surface. In a hypothetical experiment, long and thin rods are sequentially thrown onto that surface with random positions and orientations.

Among the rods that intersect the circle in two points, what is the probability P that its chord is larger than a? This question has no unique answer.

(1) All intersection points are equally likely. Therefore, we can pick rods through one intersection point with random orientations. Rods with chord $L > a$ have a restricted angle of orientation: $P = 1/3$.

(2) All directions are equally likely. Therefore, we can pick rods of one direction and random parallel displacements. Rods with chord $L > a$ have a restricted distance from the center of the circle: $P = 1/2$.

(3) The midpoint of an intersecting rod is equally likely at any point inside the circle. Rods with chord $L > a$ have midpoints with restricted distance from the center of the circle: $P = 1/4$



Conclusion: The protocol of randomization in this hypothetical experiment is insufficiently described.

Further analysis and related problem:

▷ [nex5] Probability distributions of chord lengths

▷ [nex12] Random quadratic equations