Symmetry properties [nln30]

Response function for Hermitian A is real and vanishes for t < 0:

$$\tilde{\chi}_{AA}(t) = \frac{i}{\hbar} \theta(t) \langle [A(t), A] \rangle = \tilde{\chi}'_{AA}(t) + i \tilde{\chi}''_{AA}(t).$$

Reactive part is real and symmetric:

$$\tilde{\chi}_{AA}'(t) = \frac{1}{2} \left[\tilde{\chi}_{AA}(t) + \tilde{\chi}_{AA}(-t) \right] = \frac{\imath}{2\hbar} \operatorname{sgn}(t) \langle [A(t), A] \rangle.$$

Dissipative part is imaginary and antisymmetric:

$$\tilde{\chi}_{AA}^{\prime\prime}(t) = \frac{1}{2i} \left[\tilde{\chi}_{AA}(t) - \tilde{\chi}_{AA}(-t) \right] = \frac{1}{2\hbar} \langle [A(t), A] \rangle.$$

Response function is determined by its reactive or dissipative part alone:

$$\tilde{\chi}_{AA}(t) = 2\theta(t)\tilde{\chi}'_{AA}(t) = 2i\theta(t)\tilde{\chi}''_{AA}(t).$$



Generalized susceptibility is complex:

$$\chi_{AA}(\omega) = \chi'_{AA}(\omega) + \imath \chi''_{AA}(\omega).$$

Real part is symmetric:

$$\chi'_{AA}(\omega) = \frac{1}{2} \big[\chi_{AA}(\omega) + \chi_{AA}(-\omega) \big] = \chi'_{AA}(-\omega).$$

Imaginary part is antisymmetric:

$$\chi_{AA}^{\prime\prime}(\omega) = \frac{1}{2i} \left[\chi_{AA}(\omega) - \chi_{AA}(-\omega) \right] = -\chi_{AA}^{\prime\prime}(-\omega).$$