Brownian motion: panoramic view

- Levels of contraction (horizontal)
- Modes of description (vertical)

$$\longrightarrow \quad \text{contraction} \quad \longrightarrow$$

	N-particle phase space	1-particle phase space	configuration space
relevant space	N-particle phase space	1-particle phase space	configuration space
dynamical variables	$\{\mathbf{x}_i,\, \mathbf{p}_i\}$	$\mathbf{x},\, \mathbf{p}$	$\mathbf{x}$
theoretical framework	Hamiltonian mechanics	Langevin theory	Einstein theory
... for dynamical variables	generalized Langevin equation	Langevin equation (for $dt \ll \tau_R$)	Langevin equation (for $dt \gg \tau_R$)
... for probability distribution	quant./class. Liouville equation	Fokker-Planck equation (Ornstein-Uhlenbeck process)	Fokker-Planck equation (diffusion process)

- Here dt is the time step used in the theory and τ_R is the relaxation time associated with the drag force the Brownian particle experiences.
- The generalized Langevin equation is equivalent to the Hamiltonian equation of motion for a generic classical many-body system and equivalent to the Heisenberg equation of motion for a generic quantum many-body system.