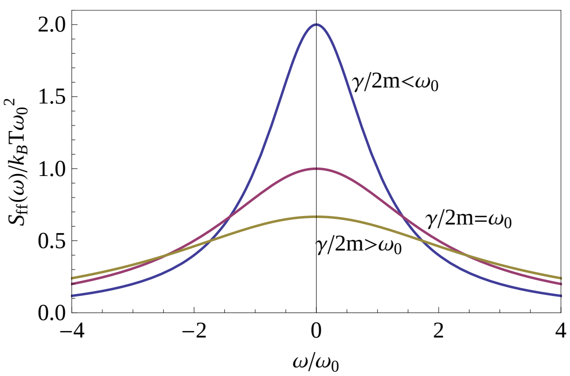
Attenuation with memory [nln22]

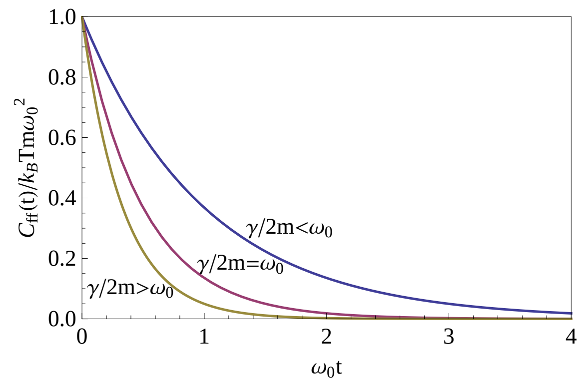
Generalized Langevin equation for Brownian harmonic oscillator:

$$m\frac{dx}{dt} + \int_{-\infty}^{t} dt'\alpha(t-t')x(t') = \frac{1}{\omega_0}f(t), \quad \alpha(t) = m\omega_0^2 e^{-(\gamma/m)t}.$$

Random force (correlated noise):

$$S_{ff}(\omega) = \frac{2k_BT\gamma m^2\omega_0^2}{\gamma^2 + m^2\omega^2}, \qquad C_{ff}(t) = k_BTm\omega_0^2 e^{-(\gamma/m)t}.$$





Stochastic variable (position):

$$S_{xx}(\omega) = \frac{2k_BT\gamma}{m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2},$$

$$C_{xx}(t) = \begin{cases} \frac{k_BT}{m\omega_0^2}\, e^{-\frac{\gamma}{2m}t}\left[\cos\omega_1 t + \frac{\gamma}{2m\omega_1}\sin\omega_1 t\right], & \omega_1 = \sqrt{\omega_0^2 - \gamma^2/4m^2} > 0 \\ \frac{k_BT}{m\omega_0^2}\, e^{-\frac{\gamma}{2m}t}\left[1 + \frac{\gamma}{2m}t\right], & \omega_0 = \gamma/2m \\ \frac{k_BT}{m\omega_0^2}\, e^{-\frac{\gamma}{2m}t}\left[\cosh\Omega_1 t + \frac{\gamma}{2m\Omega_1}\sinh\Omega_1 t\right], & \Omega_1 = \sqrt{\gamma^2/4m^2 - \omega_0^2} > 0 \end{cases}$$