Unbounded Support and Gap [nln103]

The presence of a gap in spectral densities with unbounded support is encoded in sequences of Δ_{2k-1} and Δ_{2k} that have the same growth-law exponent λ but grow with different (asymptotic) amplitudes.

Model spectral density with Gaussian decay and a gap:

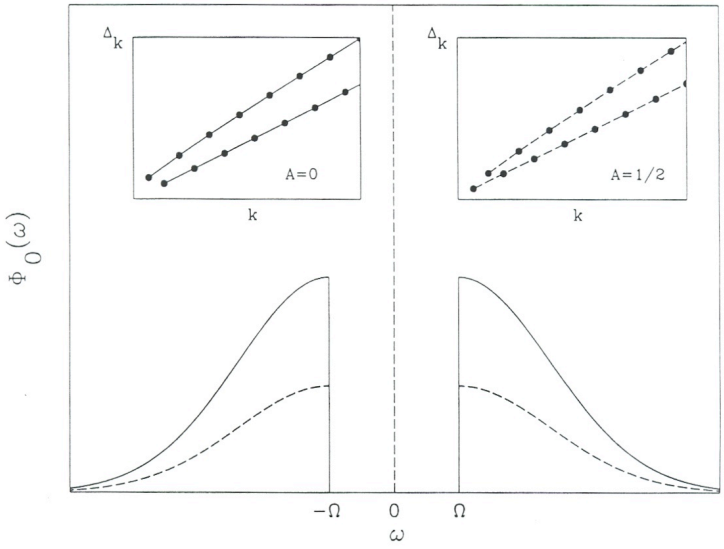
$$\Phi_0(\omega) = 2\pi A\delta(\omega) + \frac{2\sqrt{\pi}}{\omega_0}(1-A)\theta\big(|\omega|-\Omega\big)e^{-(|\omega|-\Omega)^2/\omega_0^2}. \tag{1}$$

Frequency moments:

$$\begin{aligned} M_{2k} &= 2\pi(1-A)\sum_{m=0}^{k}\left(\begin{array}{c}2k\\2m\end{array}\right)\omega_0^{2m}\Omega^{2(k-m)}2^{-m}(2m-1)!! \\ &\quad + 2\sqrt{\pi}(1-A)\sum_{m=0}^{k-1}\left(\begin{array}{c}2k\\2m+1\end{array}\right)\Omega^{2(k-m)-1}\omega_0^{2m+1}m!, \quad k=1,2,\ldots \end{aligned} \tag{2}$$

with the Δ_k to be determined from the M_{2k} as described in [nln85].

Graphical representations for two cases [Viswanath and Müller 1994]:



In the cases shown the asymptotics set in early.

- $A = 0$: Δ_{2k+1} grow more steeply: spectral density consists of continuum split by gap alone.
- $A = \frac{1}{2}$: Δ_{2k+1} grow less steeply: spectral density consists of continuum split by gap and a central spectral line.