Bandwith and Gap in Spectral Density [nln101]

Consider a model Δ_k-sequence that is periodic with period two:

$$\Delta_{2k-1} = \Delta_o, \quad \Delta_{2k} = \Delta_e, \quad k = 1, 2, \ldots \tag{1}$$

Relaxation function with this genetic code:

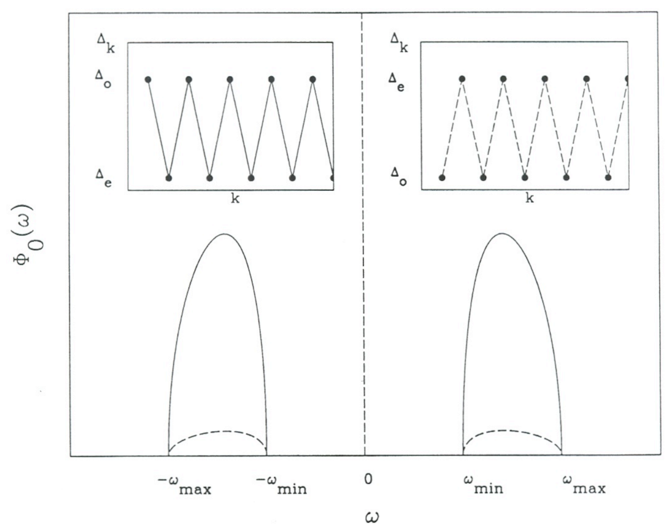
$$c_0(z) = \cfrac{1}{z + \cfrac{\Delta_o}{z + \Delta_e c_0(z)}} = \frac{1}{2\Delta_e}\left[\sqrt{2(\Delta_o + \Delta_e) + z^2 + \frac{(\Delta_o - \Delta_e)^2}{z^2}} - z - \frac{\Delta_o - \Delta_e}{z}\right]. \tag{2}$$

Spectral density has bounded support and gap:

$$\Phi_0(\omega) = \frac{1}{\Delta_e}\sqrt{2(\Delta_o + \Delta_e) - \omega^2 - \frac{(\Delta_o - \Delta_e)^2}{\omega^2}}\,\theta\big(|\omega| - \omega_{\min}\big)\theta\big(\omega_{\max} - |\omega|\big) + \frac{\pi}{\Delta_e}\Big[|\Delta_o - \Delta_e| - (\Delta_o - \Delta_e)\Big]\delta(\omega), \tag{3}$$

$$\omega_{\min} = \left|\sqrt{\Delta_o} - \sqrt{\Delta_e}\right|, \quad \omega_{\max} = \left|\sqrt{\Delta_o} + \sqrt{\Delta_e}\right|.$$

Graphical representation for two cases [Viswanath and Müller 1994]:



- $\Delta_o > \Delta_e$: continuum only (solid lines).
- $\Delta_o < \Delta_e$: continuum plus central δ-peak (dashed lines).