[tex37] **Coexistence line of continuous phase transition**

Consider 1 mol of a fluid with two phases 1 and 2 in coexistence. The coexistence line is $p(T)_{coex}$. Suppose that the volume $V$ and the entropy $S$ vary continuously at the transition ($\Delta S = 0$ and $\Delta V = 0$), but the response functions $C_p$ (heat capacity at constant pressure), $\alpha_p$ (thermal expansivity), and $\kappa_T$ (isothermal compressibility) are discontinuous. Now consider the differentials $dS$ and $dV$ for each phase and for paths in the $(T, p)$-plane. Then calculate $\Delta S \doteq dS^{(2)} - dS^{(1)}$ and $\Delta V \doteq dV^{(2)} - dV^{(1)}$ between points an infinitesimal distance across the coexistence line

(a) at constant $p$,

(b) at constant $T$.

In the limit where the distance between the two points shrinks to zero, the ratio $\Delta S/\Delta V$ stays finite and expresses (via Clausius-Clapeyron) the slope $(dp/dT)_{coex}$ of the coexistence line in terms of the discontinuities, $\Delta C_p, \Delta\alpha_p, \Delta\kappa_T$, in the response functions.

(c) Derive a relation between $\Delta C_p, \Delta\alpha_p, \Delta\kappa_T$ from the consistency condition of the results obtained in parts (a) and (b).