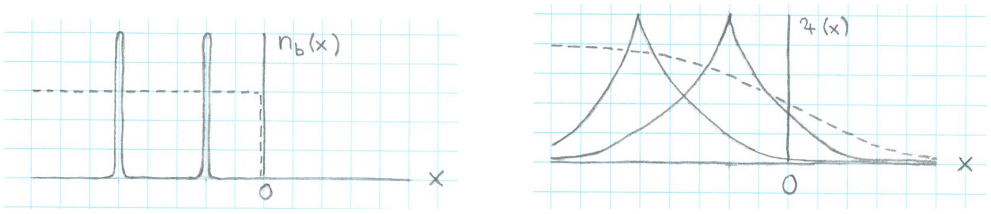
[pex8] Electric potential near interface to poly-electrolyte gel I

Here we invoke the principle of superposition to construct from the solution $\psi_l(x)$ of [pex9] for a thin layer of bound charge the solution $\psi_c(x)$ for a semi-infinite poly-electrolyte gel with a uniform density of bound charge and a flat interface at $x = 0$. The superposition principle is exact for any linear ODE. Here we use it for the linearized Poisson-Boltzmann equation.



The sketch shows (on the left) how we build up the bound charge from thin layers and (on the right) the potential for the continuum from the exponentially decaying potentials of thin layers. Carry out the integral below to find

$$\sum_{x_i<0} \psi_l(x - x_i) \to \psi_c(x) = \int_{-\infty}^{0} dx' \psi_l(x - x') = \frac{e_l n_l}{2\kappa^2 \epsilon} \left[e^{-\kappa x} \theta(x) + (2 - e^{\kappa x}) \theta(-x) \right].$$

The associated bound-charge density,

$$\sum_{x_i<0} n_l \delta(x_i) \to n_b(x) = n_l \theta(-x),$$

will be confirmed in [pex7].

Solution: