[pex36] Polymer stress relaxation: linear response

The basic model for stress relaxation expresses the time-dependent stress $\sigma(t)$ that results from a strain e_0 forced abruptly and held constant, $\sigma(t) = G(t)e_0$. For a viscoelastic material, the relaxation modulus $G(t)$ is a monotonically decreasing function that approaches zero asymptotically as $t \to \infty$. For situations with time-dependent strain $e(t)$, this linear response generalizes into the relation (Boltzmann superposition principle)

$$\sigma(t) = \int_{-\infty}^{t} d\tau\, G(t-\tau) \frac{de(\tau)}{d\tau}.$$

Here we consider two alternative relaxation moduli: one decaying exponentially and the other as a power law, representing the viscoelastic behavior of different hypothetical polymer melts:

$$G_1(t) = e^{-t}, \qquad G_2(t) = \frac{1}{1+t}.$$

(a) Calculate the time-dependent stress, $\sigma_i(t),\ i = 1, 2$, in (linear) response to a harmonically oscillating strain: $e(t) = \sin(\omega t)$ for the two model relaxation moduli. Plot $G_1(t)$ and $G_2(t)$ in the same graph for $0 < t < 5$ for comparison. Then plot $\sigma_1(t)$ and $\sigma_2(t)$ (two frames) for $0 < t < 10$ and $\omega = 0.5, 1, 2$ (three curves each). Interpret your results.

(b) Calculate the time-dependent stress, $\sigma_i(t),\ i = 1, 2$, in (linear) response to a strain that rises from zero at a constant rate: $e(t) = t$. Plot $\sigma_1(t)$ and $\sigma_2(t)$ in the same frame for $0 < t < 5$ for comparison. Interpret your results.

Solution: