[pex31] Ideal polymer chain with fixed valence angle II

The mean-square end-to-end distance of a very long, ideal polymer chain with fixed valence angle and free internal rotation angle is

$$\langle R^2 \rangle = Na^2 + 2a^2 \sum_{i=1}^{N} \sum_{k=1}^{N-i} \langle \cos\theta_{i,i+k} \rangle \overset{N\gg 1}{\rightsquigarrow} Na^2 \frac{1+\cos\gamma}{1-\cos\gamma}$$

as calculated in [pex30].
(a) Use the last expression to calculate the Kuhn segment length l_K and use the multiplicativity property, $\langle \cos\theta_{i,i+k} \rangle = (\cos\gamma)^k$, to calculate the persistent length $\tilde{l}$. The former is defined in [pex29] and the latter in [pex28].
(b) Plot l_K/a, $\tilde{l}/a$, and $l_K/\tilde{l}$ versus valence angle for $0 < \gamma < \pi/2$. Interpret your results and analyze the singularities of all three curves.

[adapted from Grosberg and Khokhlov 1994]

Solution: