[pex23] Adhesive force between flat colloidal surfaces

The (attractive) van der Waals interaction between atoms is a consequence of correlated quantum fluctuations, effectively a coupling between induced electric dipoles. The interatomic potential energy inferred from a quantum mechanical perturbation calculation reads

$$V(r) = -\frac{C}{r^6}, \quad C \doteq \frac{3}{4}\left(\frac{1}{4\pi\epsilon_0}\right)^2 \alpha^2 \hbar\omega,$$

where α is the polarizability and $\hbar\omega$ the ionisation energy. This microscopic interaction gives rise to an adhesive force between mesoscopic and macroscopic object. That adhesion plays a significant role in colloids of various shapes and sizes.

Consider the idealized situation of two semi-infinite volumes of colloidal matter (colloids 1 and 2) with flat surfaces separated by a distance h. Calculate the adhesive interaction energy between these two objects. Start from the generic expression,

$$U_{12} = -\int d^3r_1\, \rho_1(\mathbf{r}_1) \int d^3r_2\, \rho_2(\mathbf{r}_2) \frac{C}{|\mathbf{r}_1 - \mathbf{r}_2|^6},$$

where ρ_1, ρ_2 (here assumed constant) are the atomic number densities in the two colloids, respectively. Pick one atom in colloid 1 and integrate d^3r_2 using cylindrical coordinates. The result is a function of the distance between the selected atom (on colloid 1) and the surface (of colloid 2). The integration d^3r_1 over a semi-infinite slab of cross sectional area L^2 then produces a result of the form $U_{12} = L^2 u(h)$. Show that the adhesive energy per unit area can be written in the form

$$u(h) = -\frac{A_H}{12\pi h^2}, \quad A_H = \pi^2 \rho_1 \rho_2 C,$$

where A_H is known as the Hamaker constant.

[adapted from Jones 2002]

Solution: