[pex22] Colloidal regime on Earth and in space

Consider a dispersion of solid particles with diameter σ and excess density $\rho = 1\text{g/cm}^3$ in a fluid medium at room temperature. In the following we investigate two measures that demarcate a colloidal regime, $\sigma < \sigma_c$, from a granular regime, $\sigma > \sigma_c$. One measure involves the Earth's gravitational field, the other does not.
(a) The first measure uses the barometric formula for the height distribution of dispersed particles at thermal equilibrium as a criterion. Calculate the average height $\langle h \rangle$ of particles with diameter σ. In the colloidal regime we must have $\langle h \rangle > \sigma$. Estimate σ_c.
(b) For the second measure we use Brownian motion in water and the Einstein-Stokes expression for the diffusion constant. For the colloidal regime we demand that the particle diffuse a distance greater than its diameter every second (on average). Estimate σ_c again and compare with the the estimate derived from the other measure.
(c) How long (in years) would a grain with $\sigma = 1\text{mm}$ take to diffuse across the distance of its diameter when propelled by Brownian motion in water at room temperature?

[adapted from D. Frenkel in Hu and Shi 2011]

Solution: