[pex2] Nematic ordering detected via polarized optical microscopy

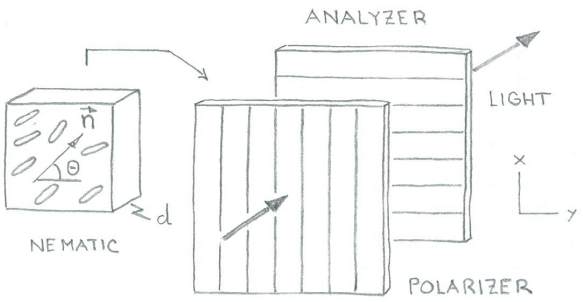
The birefringence of a nematic phase makes it detectable via polarized optical microscopy. In the simple setup sketched below consider a beam of monochromatic light of wavelength λ traveling through a slab of nematic liquid crystal with director $\mathbf{n}$ and width d as shown. The electric field of a plane wave that has made it through the polarizer is of the form

$$\mathbf{E}_1(z,t) = A\cos(kz - \omega t)\,\hat{\mathbf{j}}.$$

(a) Write the electric field vector of $\mathbf{E}_2(z,t)$ of the light wave after transmission through the slab of nematic and the electric field vector $\mathbf{E}_3(z,t)$ after the transmission through the analyzer.
(b) Show that the intensity of the emerging light wave is of the form

$$I \propto \sin^2(2\theta)\sin^2\left(\frac{\pi d}{\lambda}\Delta n\right),$$

where $\Delta n \doteq n_{\parallel} - n_{\perp}$ is the difference of refractive index for light polarized parallel or perpendicular to the director.



Solution: