[pex19] Entropy from heat capacity of glass

Consider a glass-forming substance. The melting temperature is $T_m = 237\text{K}$ with latent heat $L_m = 24.2\text{kJ/mol}$. The glass transition temperature observed in an experiment carried out on a particular time scale is $T_g^{(1)} = 160\text{K}$. The molar heat capacity is measured to jump by $\Delta C_p^{(1)} = 180\text{JK}^{-1}\text{mol}^{-1}$ between the glass and the melt in this experiment. Experiments carried out on different time scales show that the jump in heat capacity is inversely proportional to the glass transition temperature: $\Delta C_p \propto 1/T$. The heat capacity is assumed to be the same, namely $C_p^0 = 90\text{JK}^{-1}\text{mol}^{-1} = \text{const}$, in the glass ($g$) state and in the crystalline (c) phase.
(a) Construct analytic expressions for $C_p^g(T)$ and $C_p^c(T)$ under the assumption that the glass transition takes place at $T_g^{(min)} < T_g < T_m$ with $T_g^{(min)}$ to be determined later.
(b) Infer analytic expressions for the entropies $S_c(T)$ and $S_g(T)$ with $S_c(T_m^+) = S_g(T_m)$ satisfied.
(c) Plot the functions $S_c(T) - S_c(T_0)$ and $S_g(T) - S_c(T_0)$ versus T for $T_0 = 100\text{K} < T < 250\text{K}$ and three values of T_g: 140K, 160K, and 200K. Here T_0 is a convenient reference temperature to be used already in part (a).
(d) The entropy difference $S_g(T) - S_c(T)$ between the glass state and the crystalline state depends on T_g. At the Kauzmann temperature, $T_K = T_g^{(min)}$, that difference is zero. Find T_K for the case at hand.

[adapted from Jones 2002]

Solution: