[pex12] Swelling equilibrium of polymer gel II

The swelling equilibrium as established in [pex13] depends on temperature directly, via the factor $k_{\rm B}T/v_{\rm c}$, and indirectly, via the interaction parameter χ in the expression for osmotic pressure. The lattice-gas origin of that expression (see [pln32]) predicts that $\chi \sim T^{-1}$. Use this information to rewrite the condition of swelling equilibrium in the form

$$\phi^{1/3} = \frac{T}{T_1} \left[-\ln(1-\phi) - \phi - \frac{T_2}{T} \phi^2 \right], \tag{1}$$

where T_1, T_2 are independent reference temperatures, which allow us to explore the T-dependence of the swelling in a two-dimensional parameter space.

(a) Extract from Eq. (1) an explicit function $T(\phi)$.

(b) Invert that function into a function $\phi_{es}(T)$ for the special case of extreme swelling ($\phi \ll 1$).

(c) Plot a curve ϕ vs T for the general case over the range $0 \le T \le 2$ and parameter values $T_1 = 0.001, T_2 = 0.5$. Interpret the curve in the light of the function $\phi_{\rm es}(T)$, which is expected to be accurate only at low ϕ .

Solution: