[pex10] Poly-electrolyte gel: profiles of ion densities

In order to extend part (c) of [pex57] to the density profiles $n_{\pm}(x)$ of positive and negative mobile ions near the interface between gel and solution we must take into account the chemical equilibrium condition,

$$\mu_0 + k_{\rm B}T \ln n_{\pm}(x) \pm e_0 \psi(x) = \mu_0 + k_{\rm B}T \ln n_s,$$

with the assumption that far away from the bound charges of the gel we have $n_+(x) = n_-(x) = n_s = \text{const}$ enforced by the charge neutrality condition.

(a) Show that the profiles become

$$n_{\pm}(x) = n_s \, e^{\mp \beta e_0 \psi(x)}.$$

(b) Plot the profiles of $n_+(x)$ and $n_-(x)$ over the same range of x with the same parameters as in [pex57]. Choose the values for n_s , βe_0 , and $\psi(+\infty)$ judiciously.

(c) The profile $n_b(x)$ of the bound charges is reasonably well approximated by $n_-(x) - n_+(x)$ except in the immediate vicinity of the interface, where charge neutrality is not satisfied. Plot $n_b(x) \simeq n_-(x) - n_+(x)$ for the same specifications as in part (b).

Solution: