[mex229] Growth of falling raindrop

A spherical raindrop of mass density ρ_W falling through fog of mass density ρ_F accumulates mass by absorbing all fog droplets (assumed stationary) in its way. The initial radius is r_0 and the initial velocity is zero. The acceleration due to gravity is g. Air resistance is to be neglected.

(a) Relate the radial growth \dot{r} of the raindrop to its instantaneous velocity v.

(b) Construct a differential equation (nonlinear second order ODE) for the radius r of the raindrop.

(c) Show that the acceleration \dot{v} is initially equal to g and approaches the asymptotic value g/7.

(d) Plot \dot{v} versus t for $0 \le t \le 2.5$ and $\rho_W/\rho_F = 1000$, g = 10, $r_0 = 0.001$ (all in SI units).

Solution: