Distinguishable particles in shared orbitals [pln9]

Consider a system of N_A orbitals of the same energy, populated with particles from two species 1 and 2. Multiple occupancy is prohibited.

Particles 1 are distinguishable from particles 2 irrespective of placement.

Number of states with N_1 particles 1 and N_2 particles 2:

$$W(N_1, N_2) = \left(\begin{array}{c} d_1 + N_1 - 1 \\ N_1 \end{array} \right) \left(\begin{array}{c} d_2 + N_2 - 1 \\ N_2 \end{array} \right),$$

$$d_m = A_m - \sum_{m'} g_{mm'}(N_{m'} - \delta_{mm'}), \quad \mathbf{g} = \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right), \quad A_1 = A_2 = N_A.$$

The two species are not mergeable into a single species. The disappearance of the distingishable trait leads to the statistics of [pln4] with $N_1 + N_2 = N$ and $A = N_A$, which is qualitatively different.

Microstates for $A = 2$:

00
10 01, 20 02
11, 22, 12 21

Microstates for $A = 3$:

000
100 010 001, 200 020 002
110 101 011, 220 202 022, 120 102 012, 210 201 021
111, 222, 112 121 211, 122 212 221

Variations:

- If there is no limit on the occupancy of orbitals, we must use $g_{mm'} = 0$ instead.
- If particles of either species must be spaced g cells apart, we must use $g_{11} = g_{12} = g_{22} = g$ and $g_{21} = g - 1$ instead.

Note: Interchanging the values of g_{12} and g_{21} yields the same $W(N_1, N_2)$. Particles 1 and 2 may be assigned different energies or may be distinguishable by some other trait. Particles from both species are categorized as compacts.