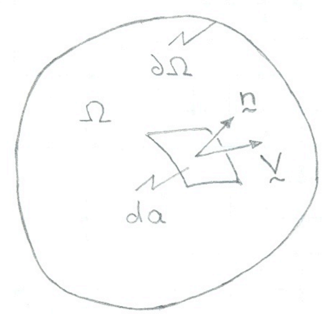
Fundamental Equations of Microfluidics I [pln85]

Gauss's theorem for generic vector field $\mathbf{V}(\mathbf{r},t)$:

$$\int_\Omega d\mathbf{r}\,\nabla\cdot\mathbf{V}(\mathbf{r},t)=\int_{\partial\Omega} da\,\mathbf{n}\cdot\mathbf{V}(\mathbf{r},t).$$

- $d\mathbf{r}\doteq d^3r$: volume element,
- da: element of surface area,
- Ω: compact region of volume,
- $\partial\Omega$: surface of that region,
- $\mathbf{n}$: surface normal (pointing outward).



Characterization of the flow of a fluid (in general, compressible) by flux densities of mass, momentum, and energy.

Mass flux:

- mass density: $\rho(\mathbf{r},t)$,
- total mass inside Ω: $M(\Omega,t)=\int_\Omega d\mathbf{r}\,\rho(\mathbf{r},t)$,
- velocity field: $\mathbf{v}(\mathbf{r},t)$,
- mass current (momentum) density: $\mathbf{J}(\mathbf{r},t)\doteq\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t)$.

Change of of mass inside Ω related to mass flux through $\partial\Omega$:

$$\frac{\partial}{\partial t}M(\Omega,t)=\int_\Omega d\mathbf{r}\,\frac{\partial}{\partial t}\rho(\mathbf{r},t)=-\int_{\partial\Omega} da\,\mathbf{n}\cdot\mathbf{J}(\mathbf{r},t)=-\int_\Omega d\mathbf{r}\,\nabla\cdot\mathbf{J}(\mathbf{r},t).$$

Local relation between mass density and mass current density implied:

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t)=-\nabla\cdot\mathbf{J}(\mathbf{r},t)\qquad\text{(continuity equation)},$$

- incompressible-fluid case:[1] $\nabla\cdot\mathbf{v}(\mathbf{r},t)=0$,
- tensor notation: $\nabla\cdot\mathbf{J}(\mathbf{r},t)=\sum_{i=x,y,z}\frac{\partial}{\partial r_i}J_i$.

[extracted from Bruus 2008]

[1] Applicable if flow velocity is small compared of speed of sound.