Particles in orbitals of different energies [pln8]

Consider a system of $N_A^{(1)}$ orbitals at energy ϵ_1 and $N_A^{(2)}$ orbitals at energy ϵ_2, populated with particles of exclusion statistics $g \geq 0$.

Particles in orbitals of different energies are (effectively) distinguishable.

Number of states with N_1 particles in orbital 1 and N_2 particles in orbital 2:

$$W(N_1, N_2) = \begin{pmatrix} d_1 + N_1 - 1 \\ N_1 \end{pmatrix} \begin{pmatrix} d_2 + N_2 - 1 \\ N_2 \end{pmatrix},$$

$$d_1 = A_1 - g(N_1 - 1), \quad d_2 = A_1 - g(N_2 - 1).$$

Generalized Pauli principle:

$$d_m = A_m - \sum_{m'} g_{mm'}(N_{m'} - \delta_{mm'}), \quad \mathbf{g} = \begin{pmatrix} g & 0 \\ 0 & g \end{pmatrix}.$$

The relation between the $N_A^{(m)}$ and the A_m is as in previous cases.

In the limit $\epsilon_1 - \epsilon_2 \to 0$, particles 1 and 2 lose their distinguishable trait. The two species can be merged: $N_1 + N_2 = N$, $A_1 + A_2 = A$:

$$\sum_{N_1=0}^{N} W(N_1, N - N_1) = W(N) = \begin{pmatrix} d + N - 1 \\ N \end{pmatrix}, \quad d = A - g(N-1).$$

Microstates for $g = 1,\ A_1 = 2,\ A_2 = 3,\ N = 3$:

11-100 11-010 11-001
10-110 10-101 10-011 01-110 01-101 01-011
00-111

Microstates for $g = 0,\ A_1 = 2,\ A_2 = 3,\ N = 2$:

20-000 11-000 02-000
10-100 10-010 10-001 01-100 01-010 01-001
00-200 00-110 00-101 00-020 00-011 00-002