# Electric Force Between Charged Plates III [pln78]

**Parallel plates with unequal charge densities:**

For this generalization of [pln72] we change the coordinate system as shown. We are looking for a solution of the linearized Poisson-Boltzmann equation,

$$\frac{d^2\psi}{dx^2} - \kappa^2\psi = 0, \quad \kappa^2 = \frac{2e_0^2 n_0}{k_B T\epsilon}, \tag{1}$$

with boundary conditions $\psi(0) = \psi_A$, $\psi(h) = \psi_B$. These boundary values are related to the charge densities on the plates via Gauss' law: $\psi'(0) = -\sigma_A/\epsilon$, $\psi'(h) = \sigma_B/\epsilon$. The solution as worked out in [pex4] reads

$$\psi(x) = \frac{\psi_A \sinh(\kappa h - \kappa x) + \psi_B \sinh(\kappa x)}{\sinh(\kappa h)}, \tag{2}$$

with

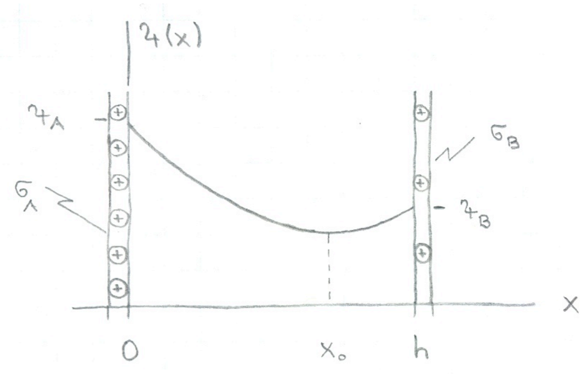
$$\psi_A = \frac{\sigma_A \cosh(\kappa h) + \sigma_B}{\kappa\epsilon \sinh(\kappa h)}, \quad \psi_B = \frac{\sigma_B \cosh(\kappa h) + \sigma_A}{\kappa\epsilon \sinh(\kappa h)}. \tag{3}$$

In generalization of [pln72] we obtain for the (repulsive) interaction force (per unit area) between the plates the expression

$$f_{\rm int} = \frac{1}{2}\epsilon\kappa^2[\psi(x_0)]^2, \quad \left.\frac{d\psi}{dx}\right|_{x_0} = 0. \tag{4}$$

This expression is evaluated in [pex3] and reads[1]

$$f_{\rm int} = \frac{1}{2}\epsilon\kappa^2\,\frac{2\psi_A\psi_B\cosh(\kappa h) - \psi_A^2 - \psi_B^2}{\sinh^2(\kappa h)}. \tag{5}$$




[adapted from Doi 2013]

[1]Positive charge densities, $\sigma_A \geq \sigma_B > 0$ constrain this expression to be positive. Hence the force will always be repulsive albeit weakened by the electrolyte.