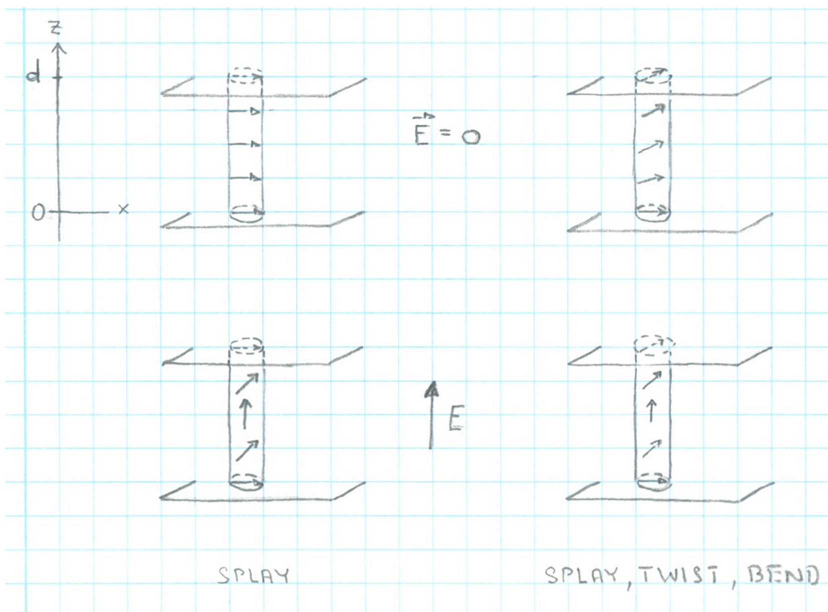
Fréedericksz Transition in LCD [pln77]

Consider a thin film of nematic sandwiched (at width d) by parallel plates and homogeneous boundary conditions with parallel alignment (left) or perpendicular alignment (right) realized. An electric field $\mathbf{E}$ perpendicular to the plane of the field is being switched on and off.



Parallel alignment:

Director field: $\mathbf{n}(z) = n_0\hat{\mathbf{e}}_x + \delta n(z)\hat{\mathbf{e}}_z$.

Boundary conditions: $\delta n(0) = \delta n(d) = 0$.

Ansatz: $\delta n(z) = A \sin\left(\frac{\pi z}{d}\right)$.

Change in energy density has two contributions:

- elastic splay energy,
- electric potential energy.

$$\Delta u(z) = \frac{1}{2}K_1\left(\frac{d\delta n}{dz}\right)^2 - \frac{1}{2}\underbrace{(\epsilon_\parallel - \epsilon_\perp)}_{\chi}\epsilon_0 E^2(\delta n)^2$$
$$= \frac{1}{2}K_1 A^2\left(\frac{\pi}{d}\right)^2\cos^2\left(\frac{\pi z}{d}\right) - \frac{1}{2}\chi\epsilon_0 E^2 A^2\sin^2\left(\frac{\pi z}{d}\right).$$

$$\Delta U(E) \doteq \int_0^d dz\, \Delta u(z) = \frac{1}{2} K_1 A^2 \left(\frac{\pi}{d}\right)^2 \frac{d}{2} - \frac{1}{2}\chi\epsilon_0 E^2 A^2 \frac{d}{2}.$$

$$\Delta U(E_\mathrm{c}) = 0 \quad \Rightarrow \quad E_\mathrm{c} = \frac{\pi}{d}\sqrt{\frac{K_1}{\chi\epsilon_0}}.$$

Perpendicular alignment:

Director field has three components: $\mathbf{n}(z) = n_x(z)\hat{\mathbf{e}}_x + n_y(z)\hat{\mathbf{e}}_y + n_z(z)\hat{\mathbf{e}}_z$.

Boundary conditions: $\mathbf{n}(0) = n_0\hat{\mathbf{e}}_x, \quad \mathbf{n}(d) = n_0\hat{\mathbf{e}}_y$.

Elastic energy now involves splay, twist, and bend.

Transition field: $E_\mathrm{c} = \dfrac{\pi}{d}\sqrt{\dfrac{K_1 + (K_3 - 2K_2)/4}{\chi\epsilon_0}}$.

Magnetic nematics:

For rod-like molecules with a magnetic dipole moment, which can be induced or permanent, the electric potential-energy density is to be replaced by the magnetic potential-energy density,

$$\Delta u(z) = -\frac{1}{2}\chi H^2 (\delta n)^2,$$

where χ is the magnetic susceptibility and H the external magnetic field.

[adapted from Jones 2002]