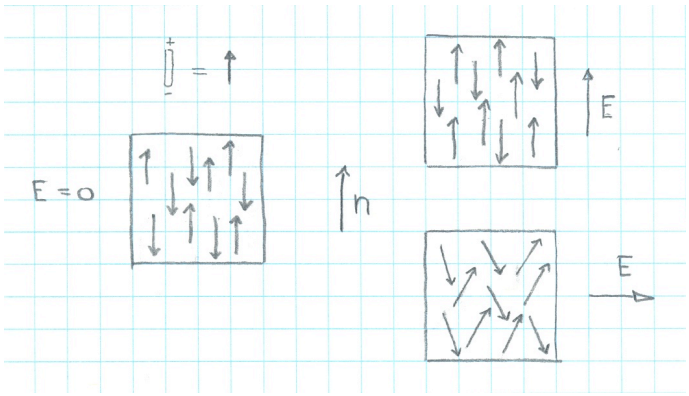
Response of Nematic to Electric Field [pln76]

Consider rod-like molecules with a permanent electric dipole moment in a nematic liquid crystal phase.

$\mathbf{E} = 0$: In the absence of an external electric field, no ferroelectric ordering is observed. The average electric dipole moment vanishes even though the nematic order parameter is nonzero. Individual molecules tend to align in one direction (director $\mathbf{n}$)[1] but the projection of the dipole moment $\mathbf{p}$ is equally like to have a positive or a negative projection onto the director: $\langle \mathbf{p} \cdot \mathbf{n} \rangle = 0$.

$\mathbf{E} \neq 0$: The presence of an external electric field produces a dielectric response in the form of an electric polarization $\mathbf{P}$ (average dipole moment). The dielectric response is anisotropic:[2]

- $\mathbf{E} \parallel \mathbf{n} \;\Rightarrow\; \mathbf{P} = (\epsilon_\parallel - 1)\epsilon_0 \mathbf{E}$ (flip response),
- $\mathbf{E} \perp \mathbf{n} \;\Rightarrow\; \mathbf{P} = (\epsilon_\perp - 1)\epsilon_0 \mathbf{E}$ (tilt response).



Displacement vector: $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$:

$$\mathbf{D} = \epsilon_\parallel \epsilon_0 (\mathbf{n} \cdot \mathbf{E})\mathbf{n} + \epsilon_\perp \epsilon_0 \big[\mathbf{E} - (\mathbf{n} \cdot \mathbf{E})\mathbf{n}\big] = \epsilon_\perp \epsilon_0 \mathbf{E} + (\epsilon_\parallel - \epsilon_\perp)\epsilon_0 (\mathbf{n} \cdot \mathbf{E})\mathbf{n},$$

- $\mathbf{E} \parallel \mathbf{n} \;\Rightarrow\; \mathbf{D} = \epsilon_\parallel \epsilon_0 \mathbf{E}$,
- $\mathbf{E} \perp \mathbf{n} \;\Rightarrow\; \mathbf{D} = \epsilon_\perp \epsilon_0 \mathbf{E}$.

Reduction in electic energy density due to polarization (with $\chi \doteq \epsilon - 1$):

$$\Delta u = -\frac{1}{2}\chi_\perp \epsilon_0 E^2 - \frac{1}{2}(\chi_\parallel - \chi_\perp)\epsilon_0 (\mathbf{E} \cdot \mathbf{n})^2.$$

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[1] Most often the director $\mathbf{n}$ is selected by boundary effects.

[2] In general, $\epsilon_\parallel > \epsilon_\perp$ is realized.