## Ion Densities Near Charged Surface [pln70]

Consider the plane interface between a solid with surface charge and a liquid with dissociated molecules. The (positive) surface charge causes an accumulation of counter-ions and a deficiency of co-ions in the vicinity.

The profile of the resulting charge density near the interface is sketched below for the cases of a weakly charged (left) and a strongly charged (right) surface.



## Analysis for weakly charged surface:

- surface charge density:  $\sigma$ ,
- ionic density in bulk:  $n_0$ ,
- monovalent ionic charge:  $e_0$ ,
- dielectric constant:  $\epsilon$ ,
- electric field near (conducting) surface:  $E = \sigma/\epsilon$ ,
- Debye screening length:  $\kappa^{-1} = \sqrt{\epsilon k_{\rm B} T / 2 n_0 e_0^2}$ .

Poisson equation in liquid (at x > 0):  $\frac{d^2}{dx^2}\psi(x) - \kappa^2\psi(x) = 0$ . One-parameter solution:  $\psi(x) = \psi(0) e^{-\kappa x}$ . Boundary condition:  $\psi'(0) = -\kappa\psi(0) = -E = -\frac{\sigma}{\epsilon}$ . Physically relevant solution:  $\psi(x) = \frac{\sigma}{\epsilon\kappa} e^{-\kappa x}$ . Ion densities:  $n_{\pm}(x) = n_0 \exp\left(\mp \frac{e_0\psi(x)}{k_{\rm B}T}\right)$  (from [pln68]). Charge density:  $\rho_e(x) = e_0 [n_+(x) - n_-(x)]$ .

[extracted from Doi 2013]