Solution of P.-B. Eq. via Fourier Transform [pln69]

For certain distributions of bound charges, $n_b(x)$, it is convenient to solve the linearized Poisson-Boltzmann equation from [pln68],

$$\frac{d^2}{dx^2}\psi(x) - \kappa^2\psi(x) = -\frac{e_b}{\epsilon}\, n_b(x), \quad \kappa^2 \doteq \frac{\beta}{\epsilon}\sum_i e_i^2 n_i^{(s)}, \tag{1}$$

via Fourier transform.[1]

Fourier transforms of electric potential and distribution of bound charge:

$$\bar{\psi}(k) \doteq \int_{-\infty}^{+\infty} dx\, e^{\imath kx}\psi(x), \quad \bar{n}_b(k) \doteq \int_{-\infty}^{+\infty} dx\, e^{\imath kx} n_b(x). \tag{2}$$

Solution of (1) in reciprocal space:

$$\bar{\psi}(k) = \frac{e_b}{\epsilon}\frac{\bar{n}_b(k)}{k^2+\kappa^2}. \tag{3}$$

One application, worked out in [pex6], considers a layer of bound charge with exponential profile of variable thickness. In the limit of a very thin layer, the result of [pex9] arrived at via a different method, can thus be reproduced.

Solution of Poisson equation for charge density $\rho_e(x)$ from [pln67] via Fourier transform:

$$\bar{\rho}_e(k) = \epsilon k^2\bar{\psi}(k) = e_b n_b(k)\frac{k^2}{k^2+\kappa^2} \xrightarrow{|k|/\kappa\to 0} 0. \tag{4}$$

Charge neutrality is satisfied at sufficiently long wavelengths. The characteristic length scale on which charge neutrality becomes accurate is the Debye screening length κ^{-1}.

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[1] This method also works when the quantities of interest vary in more than one spatial direction.