Persistence Length and Kuhn Segment Length [pln51]

More realistic models than the FJC model [pln50] take into account (i) geometric constraints and (ii) elastic energies associated with joints.

Both features introduce spatial memory of chain direction. The FJC model has no such memory beyond the distance between successive joints.

The two most common measures for spatial memory of chain direction are the *persistence length* l_p and the the *Kuhn segment length* l_K. The former has a more direct physical interpretation while the latter is, in general, easier to calculate.

Persistence length:

If s is the distance along the contour of the polymer between two (microscopic or mesoscopic) segments oriented at an angle θ between them, then $\langle \cos\theta(s)\rangle$ is a convenient measure for the mean directional change of the polymer over that distance, expected to decay exponentially as argued in [pex28]:

$$\langle \cos\theta(s)\rangle = e^{-s/l_\mathrm{p}},$$

where the persistence length l_p can be shown to grow with increasing bending stiffness in the joints and to shrink with increasing temperature due to thermal fluctuations [pex28].

Kuhn segment length:

If the polymer of contour length $L = Na$ can be conceived as a chain of N_s effectively freely jointed segments of length l_K then we have $L = N_\mathrm{s} l_\mathrm{K}$ and the mean-square end-to-end distance becomes

$$R_0^2 \doteq \langle \mathbf{r}^2\rangle = N_\mathrm{s} l_\mathrm{K}^2 = L l_\mathrm{K}.$$

The Kuhn segment length is thus defined as the ratio between the mean-square end-to-end distance and the contour length:

$$l_\mathrm{K} \doteq \frac{R_0^2}{L}.$$

In the FJC case [pln50] we have $R_0^2 = Na^2$ and $L = Na$. The Kuhn segment length assumes its minimum value, the length of a monomer: $l_\mathrm{K} = a$.