Brownian Motion of Particles with Shapes [pln40]

Langevin equation: $m\ddot{x} = -\gamma\dot{x} - \dfrac{dU}{dx} + f$.

- $m\ddot{x}$: inertia, ignorable in diffusive regime,
- $-\gamma\dot{x}$: drag force,
- $-dU/dx$: force from external potential,
- f: white-noise random force, $\langle f(t)f(t')\rangle = 2\gamma k_{\mathrm{B}}T\delta(t-t')$.

Overdamped motion: $\gamma\dfrac{dx}{dt} = -\dfrac{dU}{dx} + f$.

Particles with several degrees of freedom and external potential $U(x_1, \ldots, x_n)$ experience three kinds of generalized forces:

- conservative force: $F_i^{(\mathrm{c})} = -\partial U/\partial x_i$,
- dissipative force: $F_i^{(\mathrm{d})} = -\sum_j \gamma_{ij}\dot{x}_j$,
- random force: f_i with $\langle f_i(t)f_j(t')\rangle = 2\gamma_{ij}k_{\mathrm{B}}T\delta(t-t')$.

Attributes of generalized friction coefficients:

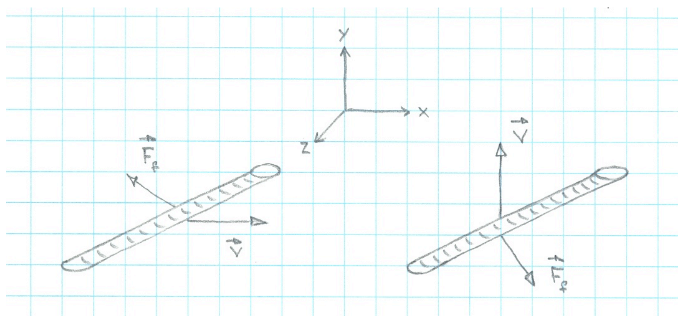
(1) $\gamma_{ij} = \gamma_{ji}$: effects are reciprocal,
(2) $\sum_{ij}\gamma_{ij}\dot{x}_i\dot{x}_j \geq 0$: dissipative work is non-negative.

In hydrodynamics, Eqs. (1) are named *Lorentz* reciprocal relations and in statistical mechanics *Onsager* reciprocal relations.

Reciprocal effects applied to rod-like particles and to particles with chirality are discussed on the next page.

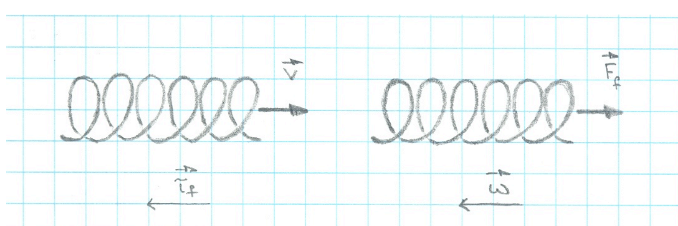
Consider a rod-like particle. When it moves in x-direction (y-direction) it experiences a drag force in the direction shown on the left (right). Reciprocity (1) implies the following relation between components of forces and velocities:

$$\frac{F_{\mathrm{f}y}}{v_x} = \frac{F_{\mathrm{f}x}}{v_y}.$$



Consider a particle with helical shape. When it moves with velocity $\vec{v}$ it experiences a torque $\vec{\tau}_{\mathrm{f}}$. When it rotates with angular velocity $\vec{\omega}$ about its axis it experiences a force $\vec{F}_{\mathrm{f}}$. Reciprocity (1) implies the following relation between (generalized) forces and velocities:

$$\frac{\tau_{\mathrm{f}}}{v} = \frac{F_{\mathrm{f}}}{\omega}.$$



[extracted from Doi 2013]