Elastic Constants [pln20]

Consider an ideal elastic material (Hookean solid).

Isotropic elastic materials are characterized by three elastic constants:

- shear modulus: $G \doteq \dfrac{\sigma}{e}, \quad e \doteq \dfrac{\Delta x}{y}, \quad \sigma \doteq \dfrac{F}{A}.$
  - ▷ σ: shear stress,
  - ▷ e: shear strain.

- bulk modulus: $K \doteq -\dfrac{\Delta p}{\Delta V/V} = \dfrac{1}{\kappa}.$
  - ▷ $\kappa \doteq -\dfrac{1}{V}\left(\dfrac{\partial V}{\partial p}\right)_{[\;]}$: isothermal $[T]$ or adiabatic $[S]$ compressibility.

- Young modulus: $Y \doteq \dfrac{\sigma}{\epsilon}, \quad \sigma \doteq \dfrac{F}{A_0}, \quad \epsilon \doteq \dfrac{\Delta L}{L_0}.$
  - ▷ σ: tensile stress,
  - ▷ ϵ: tensile strain.

Relation between elastic constants from continuum mechanics:

$$Y = \frac{9KG}{3K+G} \quad \xrightarrow{K \gg G} \quad 3G \qquad (K \gg G: \text{ incompress. mat.})$$

Poisson ratio: $\nu \doteq -\dfrac{\epsilon_\perp}{\epsilon} = \dfrac{3K-2G}{2(3K+G)} \quad \xrightarrow{K \gg G} \quad \dfrac{1}{2}, \qquad \epsilon_\perp \doteq \dfrac{\Delta L_\perp}{L_{0\perp}}.$