Population density of single species [pln17]

Statistical mechanics of single species of particles with specifications ϵ, A, g. Equations from [pln16] to be solved:

$$e^{\beta\epsilon} = w^g(1+w)^{1-g}, \quad (w+g)\langle N\rangle = A.$$

Population density of particles: $\langle N\rangle = \dfrac{A}{w+g}$.

Partition function: $Z = \left(1+w^{-1}\right)^A = \left[1 + \dfrac{\langle N\rangle}{A - g\langle N\rangle}\right]^A$.

- fermions: $g=1, \quad w = e^{\beta\epsilon}, \quad \langle N\rangle = \dfrac{A}{e^{\beta\epsilon}+1}, \quad Z = \left[1+e^{-\beta\epsilon}\right]^A.$
- bosons: $g=0, \quad w = e^{\beta\epsilon}-1, \quad \langle N\rangle = \dfrac{A}{e^{\beta\epsilon}-1}, \quad Z = \left[1-e^{-\beta\epsilon}\right]^{-A}.$
- semions: $g=\dfrac{1}{2}, \quad w = \dfrac{1}{2}\left[\sqrt{1+(2e^{\beta\epsilon})^2}-1\right], \quad \langle N\rangle = \dfrac{2A}{\sqrt{1+(2e^{\beta\epsilon})^2}},$

$$Z = \left[\frac{\sqrt{1+(2e^{\beta\epsilon})^2}+1}{\sqrt{1+(2e^{\beta\epsilon})^2}-1}\right]^A$$

Population density of bosons diverges as $\beta\epsilon \to 0^+$.

Asymptotic population density for $g>0$: $\displaystyle\lim_{\beta\epsilon\to-\infty}\langle N\rangle = \frac{A}{g}$.