Configurational entropy [pln15]

Statistically interacting particles are free of interaction energies. The energy of any many-body microstate is determined by its particle content $\{N_m\}$.

From the multiplicity of many-body microstates of given particle content $\{N_m\}$ we infer a configurational entropy for macrostates with given average particle content $\{\langle N_m\rangle\}$ [Isakov1994]:

$$S(\{\langle N_m\rangle\}) = k_B \sum_m \Big[\big(\langle N_m\rangle + Y_m\big)\ln\big(\langle N_m\rangle + Y_m\big) - \langle N_m\rangle \ln\langle N_m\rangle - Y_m \ln Y_m\Big],$$

$$Y_m \doteq A_m - \sum_{m'} g_{mm'}\langle N_{m'}\rangle.$$

The function $S(\{\langle N_m\rangle\})$ is independent of the particle energies ϵ_m. However, the particle population average $\langle N_m\rangle$ for each species does, in general, depend on the energies $\epsilon_{m'}$ of particles from all species.

Examples for one species of particles in a system with N_A orbitals. All expressions are entropies per orbital, $\bar{S} \doteq S/N_A$, as functions of population densities, $\langle\bar{N}\rangle \doteq \langle N\rangle/N_A$, in the limit $N_A \to \infty$.

- fermions: $\bar{S}/k_B = -\big(1-\langle\bar{N}\rangle\big)\ln\big(1-\langle\bar{N}\rangle\big) - \langle\bar{N}\rangle\ln\langle\bar{N}\rangle$,
- bosons: $\bar{S}/k_B = \big(1+\langle\bar{N}\rangle\big)\ln\big(1+\langle\bar{N}\rangle\big) - \langle\bar{N}\rangle\ln\langle\bar{N}\rangle$,
- semions: $\bar{S}/k_B = \left(1+\frac{1}{2}\langle\bar{N}\rangle\right)\ln\left(1+\frac{1}{2}\langle\bar{N}\rangle\right) - \langle\bar{N}\rangle\ln\langle\bar{N}\rangle - \left(1-\frac{1}{2}\langle\bar{N}\rangle\right)\ln\left(1-\frac{1}{2}\langle\bar{N}\rangle\right)$,