Hosts, hybrids, and tags [pln13]

Here we generalize [pln12] by assigning different energies, ϵ_1, to the first particle (host) placed into any orbital, ϵ_2, to the second particle (hybrid) placed into the same orbital, and ϵ_3, to all further particles (tags).

Multiplicity of microstates with N_1 hosts, N_2 hybrids, and N_3 tags:

$$W(\{N_m\}) = \prod_{m=1}^{3} \left(\begin{array}{c} d_m + N_m - 1 \\ N_m \end{array} \right), \quad d_m = A_m - \sum_{m'} g_{mm'}(N_{m'} - \delta_{mm'}),$$

$$\mathbf{g} = \left(\begin{array}{ccc} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{array} \right), \quad A_1 = N_A, \quad A_2 = A_3 = 0.$$

Here the code n of an orbital state means vacant ($n = 0$), contains a host ($n \geq 1$), contains a host and a hybrid ($n \geq 2$), contains a host, a hybrid, and $n - 2$ tags ($n \geq 3$).

Microstates for $N_A = 2$ and $N_1 + N_2 + N_3 \leq 5$:

00
10 01
20 02, 11
30 03, 21 12
40 04, 31 13, 22
50 05, 41 14, 32 23

Microstates for $N_A = 3$ and $N_1 + N_2 + N_3 \leq 5$:

000
100 010 001
200 020 002, 110 101 011
300 030, 003, 210 201 021 120 102 012, 111
400 040 004, 220 202 022, 310 103 031 130 301 013, 211 121 112
500 050 005, 410 104 041 140 401 014, 230 302 023 320 203 032,
221 212 122, 311 131 113