Hosts and tags [pln12]

Here we consider a variation of [pln10]: a system of N_A orbitals where each orbital may be vacant, singly occupied, or multiply occupied with no limit.

The first particle placed into an orbital belongs to the *host* category and has activation energy ϵ_1. All further particles placed into the same orbital belong to the *tag* category and have activation energy ϵ_2.

Hosts and tags become indistinguishable if $\epsilon_1 = \epsilon_2$. Then they are mergeable into bosons as in [pln5].

Multiplicity of microstates with N_1 hosts and N_2 tags:

$$W(N_1, N_2) = \left(\begin{array}{c} d_1 + N_1 - 1 \\ N_1 \end{array} \right) \left(\begin{array}{c} d_2 + N_2 - 1 \\ N_2 \end{array} \right),$$

$$d_m = A_m - \sum_{m'} g_{mm'}(N_{m'} - \delta_{mm'}), \quad \mathbf{g} = \left(\begin{array}{cc} 1 & 0 \\ -1 & 0 \end{array} \right), \quad A_1 = N_A, \quad A_2 = 0.$$

The system without particles has only capacity for hosts ($A_1 = N_A$), not for tags ($A_2 = 0$). Adding hosts decreases capacity for further hosts ($g_{11} = 1$), but increases capacity for tags ($g_{21} = -1$). Adding tags does not affect capacity for hosts or further tags ($g_{12} = g_{22} = 0$).

We encode the occupancy of each orbital as vacant (0) or accommodating one host and $n - 1$ tags (n).

Microstates for $N_A = 2$ and $N_1 + N_2 \leq 4$:

00
10 01
20 02, 11
30 03, 21 12
40 04, 31 13 22

Microstates for $N_A = 3$ and $N_1 + N_2 \leq 5$:

000
100 010 001
200 020 002, 110 101 011
300 030, 003, 210 201 021 120 102 012, 111
400 040 004, 220 202 022 310 103 031 130 301 013, 211 121 112
500 050 005, 410 104 041 140 401 014 230 302 023 320 203 032,
221 212 122, 311 131 113