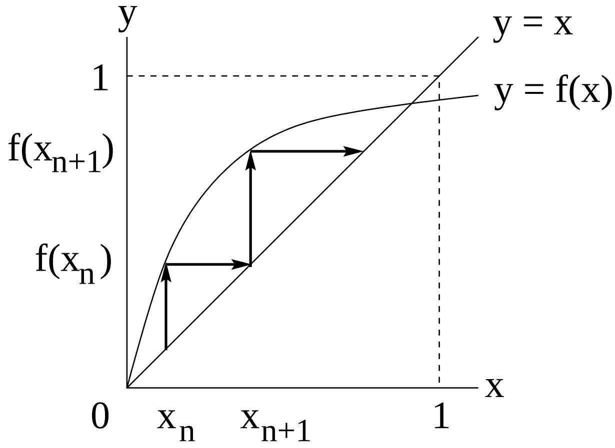
Iterated Maps [mln34]

An *iterated map* is specified by a continuous function $f(x)$ and generates from a given initial value x_0 a sequence of values $x_1 = f(x_0), x_2 = f(x_1), \ldots$.

Graphical representation of the iteration $x_{n+1} = f(x_n)$:



Fixed points: Solutions of $x = f(x)$. Some are attractors, others are repellors.

The *logistic map* is an iterated map that depends on a continuous parameter A. It is obtained by replacing the *differential equation*, $dx/dt = Ax(1-x)$, of the continuous logistic model [mln32] with a *difference equation*:

$$x_{n+1} = Ax_n\,(1-x_n)\,, \quad x_n \in [0,1].$$

The logistic map has fixed points at $x = 0$ and (if $A > 1$) at $x = 1 - 1/A$. Depending on the value of A, either one can be an attractor or a repellor.

Depending on the values of A, continued iteration produces sequences $x_1, x_2, \ldots$ with different asymptotic properties:

- $x_{n+1} - x_n \to 0$ (point attractor),
- $x_{n+m} - x_n \to 0$ for fixed $m \geq 2$ (period-m limit cycle),
- $x_{n+1} - x_n$ without recognizable pattern (chaos).

A variation of the parameter A produces transitions between the three kinds of asymptotic behavior.

Bifurcation diagram: Plot of asymptotic values x_n versus parameter A.