[tex45] Thermodynamics of the mean−field ferromagnet I

The mean-field ferromagnet is specified by the heat capacity $C_M = 0$ and by the equation of state $M = \tanh([H + \lambda M]/T)$, where λ is a constant. In zero magnetic field ($H = 0$), this system undergoes a continuous transition at temperature $T_c = \lambda$ between a paramagnetic phase ($M = 0$) and a ferromagnetic phase ($M \neq 0$).
(a) Determine the spontaneous magnetization $M(T, H = 0)$ in the ferromagnetic phase by numerically solving the equation of state at $H = 0$. Plot M versus T for $0 \leq T \leq T_c$.
(b) Show that the entropy depends only on M:

$$S(M) = -\frac{1+M}{2} \ln \frac{1+M}{2} - \frac{1-M}{2} \ln \frac{1-M}{2}.$$

Plot S versus T at $H = 0$ for $0 \leq T \leq 2T_c$.
(c) Calculate an analytic expression for the Helmholtz free energy $A(T, M)$.

Solution: