

**[tex169] Ultrarelativistic classical ideal gas (grandcanonical ensemble)**

Consider an ultrarelativistic classical ideal gas,

$$\mathcal{H}_N = \sum_{l=1}^N |\mathbf{p}_l|c,$$

within a region of volume  $V$  in equilibrium in contact with heat reservoir at temperature  $T$  and a particle reservoir at chemical potential  $\mu$ .

- (a) Calculate the grand partition function  $Z(T, V, z)$ , where  $z = \exp(\mu/k_B T)$  is the fugacity.
- (b) Derive from  $Z$  the grand potential  $\Omega(T, V, \mu)$ , the entropy  $S(T, V, \mu)$ , the pressure  $p(T, V, \mu)$ , and the average particle number  $\langle N \rangle = \mathcal{N}(T, V, \mu)$ .
- (c) Derive from these expressions the familiar results for the internal energy  $U$ , and the ideal gas equation of state,  $pV = \mathcal{N}k_B T$ .

**Solution:**